Project 2: GPS, Conditioning, and Nonlinear squares solutions
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MATLAB Code

Problem 1 Functions / Scripts

```
function out = P2_prob4_newton(x0,y0,z0,d0,sat1,sat2,sat3,sat4)
%Function that was used for solving the roots for problem 1, and
reused for
%the new satelites in problems 4 and 5, and slightly modified in
problem 6
%to take into account for the increased number of satelites, and
solving
%the equation with the Guass-Newton iteration.

%Input includes intial location guesses, and also 4 satelite vectors
with
%their appropriate locations (x,y,z,t) so they can be indexed later on
cc = 299792.458;

A1=sat1(1);
B1=sat1(2);
C1=sat1(3);
t1=sat1(4);

A2=sat2(1);
B2=sat2(2);
C2=sat2(3);
t2=sat2(4);

A3=sat3(1);
B3=sat3(2);
C3=sat3(3);
t3=sat3(4);

A4=sat4(1);
B4=sat4(2);
C4=sat4(3);
t4=sat4(4);

u = [x0,y0,z0,d0];
for i = 1:4
    x = u(1);
    y = u(2);
    z = u(3);
    d = u(4);
    F1 = (x - A1).^2 + (y - B1).^2 + (z - C1).^2 - (cc*(t1 - d)).^2;
    F2 = (x - A2).^2 + (y - B2).^2 + (z - C2).^2 - (cc*(t2 - d)).^2;
    F3 = (x - A3).^2 + (y - B3).^2 + (z - C3).^2 - (cc*(t3 - d)).^2;
    F4 = (x - A4).^2 + (y - B4).^2 + (z - C4).^2 - (cc*(t4 - d)).^2;
    FX = [F1;F2;F3;F4];
    dx1 = 2*(x-A1);
    dy1 = 2*(y-B1);
    dz1 = 2*(z-C1);
```

```
    dd1 = 2*cc*(cc*t1-cc*d);

    dx2 = 2*(x-A2);
    dy2 = 2*(y-B2);
    dz2 = 2*(z-C2);
    dd2 = 2*cc*(cc*t2-cc*d);

    dx3 = 2*(x-A3);
    dy3 = 2*(y-B3);
    dz3 = 2*(z-C3);
    dd3 = 2*cc*(cc*t3-cc*d);

    dx4 = 2*(x-A4);
    dy4 = 2*(y-B4);
    dz4 = 2*(z-C4);
    dd4 = 2*cc*(cc*t4-cc*d);
    DF = [dx1 dy1 dz1 dd1;
          dx2 dy2 dz2 dd2;
          dx3 dy3 dz3 dd3;
          dx4 dy4 dz4 dd4];
    v = DF\FX;
    u(1) = u(1) - v(1);
    u(2) = u(2) - v(2);
    u(3) = u(3) - v(3);
    u(4) = u(4) - v(4);
end
out = u;
```

Problem 2 Functions / Scripts

```
%P2_2
%Problem 2 Script
cc = 299792.458;
A1=15600;
B1=7540;
C1=20140;
t1=0.07074;

A2=18760;
B2=2750;
C2=18610;
t2=0.07220;

A3=17610;
B3=14630;
C3=13480;
t3=0.07690;

A4=19170;
B4=610;
C4=18390;
t4=0.07242;
```

```
row1 = [(2*(A2-A1)),(2*(B2-B1)),(2*(C2-C1)),(2*cc^2*(t1-t2)),(A2^2-
A1^2+B2^2-B1^2+C2^2-C1^2+(cc^2*t1^2)-(cc^2*t2^2))];
row2 = [(2*(A3-A1)),(2*(B3-B1)),(2*(C3-C1)),(2*cc^2*(t1-t3)),(A3^2-
A1^2+B3^2-B1^2+C3^2-C1^2+(cc^2*t1^2)-(cc^2*t3^2))];
row3 = [(2*(A4-A1)),(2*(B4-B1)),(2*(C4-C1)),(2*cc^2*(t1-t4)),(A4^2-
A1^2+B4^2-B1^2+C4^2-C1^2+(cc^2*t1^2)-(cc^2*t4^2))];

matrix = [row1;row2;row3];
matricksrforkids = rref(matrix);

r14 = matricksrforkids(1,4);
r15 = matricksrforkids(1,5);
r24 = matricksrforkids(2,4);
r25 = matricksrforkids(2,5);
r34 = matricksrforkids(3,4);
r35 = matricksrforkids(3,5);

a = ((r14^2)+(r24^2)+(r34^2)-(cc^2));
b = 2*((-r15*r14)+(A1*r14)-(r25*r24)+(B1*r24)-
(r35*r34)+(C1*r34)+(cc*cc*t1));
c = ((r15^2)-(2*A1*r15)+(A1^2)+(r25^2)-(2*B1*r25)+(B1^2)+(r35^2)-
(2*C1*r35)+ (C1^2)-(cc^2*t1^2));

d = (-b + sqrt(b^2-(4*a*c)))/(2*a);
```

Problem 4 and 5 Functions / Scripts

```
function out = prob4(phi,theta)
%Function that generates a new satelite based on phi and theta
rho = 26570;
cc = 299792.458;

Ai = rho * cos(phi) * cos(theta);
Bi = rho * cos(phi) * sin(theta);
Ci = rho * sin(phi);
x=0;
y=0;
z=6370;
d=0.0001;
% Ri used to find time (t)
Ri = sqrt((Ai-x)^2 + (Bi-y)^2 + (Ci - z)^2);
Ti = d + Ri/cc;
%output is the location and transmission time of the new satelite as
an
%array so that it can easily be indexed in other functions
out = [Ai,Bi,Ci,Ti]';

function out = prob4_errors(sat1,sat2,sat3,sat4)
%Function that takes in four satelites, generates the initial location
of
%the receiver, modifies the transmission times of the satelites, and
then
```

```
%recalculates the location of the receiver to calculate the forward
error,
%backward error and EMF

x0 = 100; y0 = 100; z0 = 6000; d0 = 0;
initial = P2_prob4_newton(x0,y0,z0,d0,sat1,sat2,sat3,sat4);

%sat1(4) = sat1(4) + 10e-8;sat2(4)=sat2(4) + 10e-8; sat3(4) = sat3(4)
+ 10e-8; sat4(4) = sat4(4) - 10e-8;
%sat1(4) = sat1(4) + 10e-8;sat2(4)=sat2(4) + 10e-8; sat3(4) = sat3(4)
- 10e-8; sat4(4) = sat4(4) - 10e-8;
%sat1(4) = sat1(4) + 10e-8;sat2(4)=sat2(4) - 10e-8; sat3(4) = sat3(4)
- 10e-8; sat4(4) = sat4(4) - 10e-8;
%sat1(4) = sat1(4) - 10e-8;sat2(4)=sat2(4) + 10e-8; sat3(4) = sat3(4)
- 10e-8; sat4(4) = sat4(4) - 10e-8;

%sat1(4) = sat1(4) - 10e-8;sat2(4)=sat2(4) + 10e-8; sat3(4) = sat3(4)
+ 10e-8; sat4(4) = sat4(4) - 10e-8;
%sat1(4) = sat1(4) - 10e-8;sat2(4)=sat2(4) + 10e-8; sat3(4) = sat3(4)
+ 10e-8; sat4(4) = sat4(4) + 10e-8;
%sat1(4) = sat1(4) - 10e-8;sat2(4)=sat2(4) - 10e-8; sat3(4) = sat3(4)
+ 10e-8; sat4(4) = sat4(4) + 10e-8;
sat1(4) = sat1(4) - 10e-8;sat2(4)=sat2(4) - 10e-8; sat3(4) = sat3(4) -
10e-8; sat4(4) = sat4(4) + 10e-8;

final = P2_prob4_newton(x0,y0,z0,d0,sat1,sat2,sat3,sat4);


dx = final(1) - initial(1);
dy = final(2) - initial(2);
dz = final(3) - initial(3);
delta = [dx dy dz];
delta
cc = 299792.458;
RFE = max(abs(delta))
RBE = cc*max(abs(10e-8));
EMF = RFE / RBE;
out = [EMF,RFE];
```

Problem 6Functions / Scripts

```
function out =
Problem6a(x0,y0,z0,d0,sat1,sat2,sat3,sat4,sat5,sat6,sat7)
%same as the original newton method, except now it takes a gauss-
newton
%approach, and takes in input from 7 satelites instead.
cc = 299792.458;

A1=sat1(1);
B1=sat1(2);
C1=sat1(3);
t1=sat1(4);

A2=sat2(1);
B2=sat2(2);
```

```
C2=sat2(3);
t2=sat2(4);

A3=sat3(1);
B3=sat3(2);
C3=sat3(3);
t3=sat3(4);

A4=sat4(1);
B4=sat4(2);
C4=sat4(3);
t4=sat4(4);

A5=sat5(1);
B5=sat5(2);
C5=sat5(3);
t5=sat5(4);


A6=sat6(1);
B6=sat6(2);
C6=sat6(3);
t6=sat6(4);

A7=sat7(1);
B7=sat7(2);
C7=sat7(3);
t7=sat7(4);

u = [x0,y0,z0,d0];
for i = 1:100
    x = u(1);
    y = u(2);
    z = u(3);
    d = u(4);
    F1 = (x - A1).^2 + (y - B1).^2 + (z - C1).^2 - (cc*(t1 - d)).^2;
    F2 = (x - A2).^2 + (y - B2).^2 + (z - C2).^2 - (cc*(t2 - d)).^2;
    F3 = (x - A3).^2 + (y - B3).^2 + (z - C3).^2 - (cc*(t3 - d)).^2;
    F4 = (x - A4).^2 + (y - B4).^2 + (z - C4).^2 - (cc*(t4 - d)).^2;
    F5 = (x - A5).^2 + (y - B5).^2 + (z - C5).^2 - (cc*(t5 - d)).^2;
    F6 = (x - A6).^2 + (y - B6).^2 + (z - C6).^2 - (cc*(t6 - d)).^2;
    F7 = (x - A7).^2 + (y - B7).^2 + (z - C7).^2 - (cc*(t7 - d)).^2;
    FX = [F1;F2;F3;F4;F5;F6;F7];
    dx1 = 2*(x-A1);
    dy1 = 2*(y-B1);
    dz1 = 2*(z-C1);
    dd1 = 2*cc*(cc*t1-cc*d);

    dx2 = 2*(x-A2);
    dy2 = 2*(y-B2);
    dz2 = 2*(z-C2);
    dd2 = 2*cc*(cc*t2-cc*d);
```

```
    dx3 = 2*(x-A3);
    dy3 = 2*(y-B3);
    dz3 = 2*(z-C3);
    dd3 = 2*cc*(cc*t3-cc*d);

    dx4 = 2*(x-A4);
    dy4 = 2*(y-B4);
    dz4 = 2*(z-C4);
    dd4 = 2*cc*(cc*t4-cc*d);

    dx5 = 2*(x-A5);
    dy5 = 2*(y-B5);
    dz5 = 2*(z-C5);
    dd5 = 2*cc*(cc*t5-cc*d);

    dx6 = 2*(x-A6);
    dy6 = 2*(y-B6);
    dz6 = 2*(z-C6);
    dd6 = 2*cc*(cc*t6-cc*d);

    dx7 = 2*(x-A7);
    dy7 = 2*(y-B7);
    dz7 = 2*(z-C7);
    dd7 = 2*cc*(cc*t7-cc*d);

    DF = [dx1 dy1 dz1 dd1;
          dx2 dy2 dz2 dd2;
          dx3 dy3 dz3 dd3;
          dx4 dy4 dz4 dd4;
          dx5 dy5 dz5 dd5;
          dx6 dy6 dz6 dd6;
          dx7 dy7 dz7 dd7;];

    D=DF'*DF;

    v = D\(DF'*FX);

    u(1) = u(1) - v(1);
    u(2) = u(2) - v(2);
    u(3) = u(3) - v(3);
    u(4) = u(4) - v(4);
end
out = u;


function out = prob6_errors(sat1,sat2,sat3,sat4,sat5,sat6,sat7)
x0 = 100; y0 = 100; z0 = 6000; d0 = 0;
%Same as problem 4 errors, but takes in 7 satelites
initial = Problem6a(x0,y0,z0,d0,sat1,sat2,sat3,sat4,sat5,sat6,sat7);
```

```
%sat1(4) = sat1(4) + 10e-8;sat2(4)=sat2(4) + 10e-8; sat3(4) = sat3(4)
+ 10e-8; sat4(4) = sat4(4) - 10e-8; sat5(4) = sat5(4) + 10e-8;sat6(4)
= sat6(4) + 10e-8;sat7(4) = sat7(4) + 10e-8;
%sat1(4) = sat1(4) + 10e-8;sat2(4)=sat2(4) + 10e-8; sat3(4) = sat3(4)
- 10e-8; sat4(4) = sat4(4) - 10e-8; sat5(4) = sat5(4) + 10e-8;sat6(4)
= sat6(4) + 10e-8;sat7(4) = sat7(4) - 10e-8;
%sat1(4) = sat1(4) + 10e-8;sat2(4)=sat2(4) - 10e-8; sat3(4) = sat3(4)
- 10e-8; sat4(4) = sat4(4) - 10e-8; sat5(4) = sat5(4) + 10e-8;sat6(4)
= sat6(4) - 10e-8;sat7(4) = sat7(4) + 10e-8;
%sat1(4) = sat1(4) - 10e-8;sat2(4)=sat2(4) + 10e-8; sat3(4) = sat3(4)
- 10e-8; sat4(4) = sat4(4) - 10e-8; sat5(4) = sat5(4) + 10e-8;sat6(4)
= sat6(4) - 10e-8;sat7(4) = sat7(4) - 10e-8;

%sat1(4) = sat1(4) - 10e-8;sat2(4)=sat2(4) + 10e-8; sat3(4) = sat3(4)
+ 10e-8; sat4(4) = sat4(4) - 10e-8; sat5(4) = sat5(4) - 10e-8;sat6(4)
= sat6(4) + 10e-8;sat7(4) = sat7(4) + 10e-8;
%sat1(4) = sat1(4) - 10e-8;sat2(4)=sat2(4) + 10e-8; sat3(4) = sat3(4)
+ 10e-8; sat4(4) = sat4(4) + 10e-8; sat5(4) = sat5(4) - 10e-8;sat6(4)
= sat6(4) + 10e-8;sat7(4) = sat7(4) - 10e-8;
%sat1(4) = sat1(4) - 10e-8;sat2(4)=sat2(4) - 10e-8; sat3(4) = sat3(4)
+ 10e-8; sat4(4) = sat4(4) + 10e-8; sat5(4) = sat5(4) - 10e-8;sat6(4)
= sat6(4) - 10e-8;sat7(4) = sat7(4) + 10e-8;
sat1(4) = sat1(4) - 10e-8;sat2(4)=sat2(4) - 10e-8; sat3(4) = sat3(4) -
10e-8; sat4(4) = sat4(4) + 10e-8; sat5(4) = sat5(4) - 10e-8;sat6(4) =
sat6(4) - 10e-8;sat7(4) = sat7(4) - 10e-8;

final = Problem6a(x0,y0,z0,d0,sat1,sat2,sat3,sat4,sat5,sat6,sat7);

dx = final(1) - initial(1);
dy = final(2) - initial(2);
dz = final(3) - initial(3);
delta = [dx dy dz];
delta
cc = 299792.458;
RFE = max(abs(delta))
RBE = cc*max(abs(10e-8));
EMF = RFE / RBE;
out = [EMF,RFE];
```

General Script for Problems 4,5,6

```
%Problem 4 EMF's and maxDistances stored in an array
condition(1) = 4.0251; maxDistance(1) = 0.1207; %+++-
condition(2) = 1.3453; maxDistance(2) = 0.0403; %++--
condition(3) = 1.3453; maxDistance(3) = 0.0403; %+---
condition(4) = 2.0238; maxDistance(4) = 0.0607; %-+--
condition(5) = 3.3982; maxDistance(5) = 0.1019; %-++-
condition(6) = 1.3453; maxDistance(6) = 0.0403; %-+++
condition(7) = 1.3453; maxDistance(7) = 0.0403; %--++
condition(8) = 4.0252; maxDistance(8) = 0.1207; %---+

maxDistanceFinal = max(abs(maxDistance)) *1000;
```

```
maxCondition = max(abs(condition));

%problem 5
%calculation of thetas and phis within 5% of each other (later used as
%direct input for generating satelites
theta(1) = pi;
theta(2) = pi + (0.01)*pi;
theta(3) = pi + (0.02)*pi;
theta(4) = pi - (0.01)*pi;

phi(1) = pi/4;
phi(2) = pi/4 + (0.01)*(pi/4);
phi(3) = pi/4 + (0.02)*(pi/4);
phi(4) = pi/4 - (0.01)*(pi/4);


news1 = prob4(phi(1),theta(1));
news2 = prob4(phi(2),theta(2));
news3 = prob4(phi(3),theta(3));
news4 = prob4(phi(4),theta(4));

%Problem 5's EMF's and changes in distance
condition2(1) = 262652.87129797; maxDistance2(1)= 7874.13498871761;
%+++-
condition2(2) = 1944.59790913057; maxDistance2(2)= 58.2975786999913;
%++--
condition2(3) = 2508704.80298543; maxDistance2(3)= 75209.0779283407;
%+---
condition2(4) = 622063.475367196; maxDistance2(4)= 18648.9938312354;
%-+--
condition2(5) = 373199.248773208; maxDistance2(5)= 11188.2320113474;
%-++-
condition2(6) = 616016.829333332; maxDistance2(6)= 18467.7199435206;
%-+++
condition2(7) = 1941.76407999955; maxDistance2(7)= 58.2126226399173;
%--++
condition2(8) = 192451.801788258; maxDistance2(8) =
5769.55987046307;%---+

maxDistanceFinal2 = max(abs(maxDistance2)); %in KM
maxCondition2 = max(abs(condition2));
minDistanceFinal = min(abs(maxDistance2));
minCondition2 = min(abs(condition2));

%Problem 6 errors
c6(1)=0.759842298379878; m6(1)=0.0227794990323673; %+++-+++
c6(2)=0.922787675792021; m6(2)=0.0276644785537797; %++--++-
c6(3)=1.52055005106692;  m6(3)=0.0455849437321376; %+---+-+
c6(4)=1.02567632076965;  m6(4)=0.0307490025315929; %-+--+--
c6(5)=1.05206993372837;  m6(5)=0.0315402631420325; %-++--++
c6(6)=1.52055075735373;  m6(6)=0.0455849649060838; %-+++-+-
c6(7)=0.922789027717949; m6(7)=0.0276645190834994; %--++--+
```

```
c6(8)=0.759843349967869; m6(8)=0.0227795305581822; %---+---

maxDistanceFinal6 = max(abs(m6)) *1000;
maxCondition6 = max(abs(c6));
```

Console / Interactions Tab

Problem 1

```
>> format long g

>> sat1=[15600,7540,20140,0.07074];

>> sat2=[18760,2750,18610,0.07220];

>> sat3=[17610,14630,13480,0.07690];

>> sat4=[19170,610,18390,0.07242];

>> P2_newton(0,0,6370,0,sat1,sat2,sat3,sat4)

ans =

 -41.7727095708001         -16.7891941065067          6370.05955922335
-0.00320156582959401


>> P2_newton(0,0,50000,0,sat1,sat2,sat3,sat4)

ans =

-39.7478181318263         -134.275259321179         -9413.77434469353
0.185174934816319

>> P2_newton(0,0,200000,0,sat1,sat2,sat3,sat4)

ans =

-43.8899701060882          106.056211088366          22873.9016780002
-0.200170997699297
```

Problem 4

```
s1 = prob4(0,0)

s2 = prob4(pi/8,pi/2)

s3 = prob4(3*pi/8,pi)

s4 = prob4(pi/2, 3*pi/2)

s1 =

                    26570

                        0
```

```
                         0

        0.0912394412763415

s2 =

      1.50309959017421e-12

          24547.4791788249

          10167.8987979404

        0.0829557883230089

s3 =

         -10167.8987979404

       1.2452084716952e-12

          24547.4791788249

        0.0695748363497502

s4 =

     -2.98864630689942e-28

     -1.62694327266726e-12

                   26570

        0.0674799472300267


>> P2_newton(100,100,6000,0,s1,s2,s3,s4)


ans =

3.92671203236846e-12      8.98330528019349e-12
6370.00000000001      0.000100100000000024

s1 = prob4(0,0)

s2 = prob4(pi/8,pi/2)

s3 = prob4(3*pi/8,pi)

s4 = prob4(pi/2, 3*pi/2)
```

Prob 4

```
maxDistance

maxDistance =

  120.7000
```

```
>> maxCondition

maxCondition =

    4.0252

>>
```

Prob 5

```
>> format long g
>> news1 = prob4(phi(1),theta(1))
news1 =
         -18787.8271761266
      2.30084524141771e-12
          18787.8271761266
        0.0752211878991392
>> news2 = prob4(phi(2),theta(2))
news2 =
         -18630.4924209237
         -585.486811338579
          18934.8054478427
        0.0750823876898929
>> news3 = prob4(phi(3),theta(3))
news3 =
         -18453.9164188573
         -1161.02201101907
          19080.6157315679
        0.0749444361336467
>> news4 = prob4(phi(4),theta(4))
news4 =
         -18925.4622643676
          594.756612113417
          18639.6899827316
        0.0753608235279171
```

```
>> P2_prob4_newton(100,100,6000,0,news1,news2,news3,news4)

ans =

-5.14652601554857e-08      2.02529624382319e-07
6370.00000073448      0.000100000001494096

>> maxDistanceFinal

maxDistanceFinal =

  120.7000

>> maxCondition

maxCondition =

    4.0252

>> format long g

>> maxDistanceFinal2

maxDistanceFinal2 =

          75209.0779283407

>> maxCondition2

maxCondition2 =

          2508704.80298543

>> minDistanceFinal

minDistanceFinal =

          58.2126226399173

>> minCondition2

minCondition2 =

          1941.76407999955
```

Problem 6

```
s5 = prob4(0,3*pi/2)

s6 = prob4(pi/2,0)

s7 = prob4(pi/8,pi)
```

```
s5 = prob4(0,3*pi/2)

s5 =

     -4.88082981800178e-12

                    -26570

                         0

        0.0912394412763415

>> s6 = prob4(pi/2,0)

s6 =

      1.62694327266726e-12

                         0

                     26570

        0.0674799472300267

>> s7 = prob4(pi/8,pi)

s7 =

         -24547.4791788249

      3.00619918034842e-12

          10167.8987979404

        0.0829557883230089

>> >> Problem6a(100,100,6000,0,s1,s2,s3,s4,s5,s6,s7)

ans =

-2.9005135546505e-12      1.29432458379864e-12
6370      9.99999999999985e-05

prob6_errors(s1,s2,s3,s4,s5,s6,s7)

>> conditionNumber

>> maxDistanceFinal6

maxDistanceFinal6 =

          45.5849649060838

>> maxCondition6

maxCondition6 =

          1.52055075735373
```